3. MEASUREMENT ON VOLTAGE DIVIDER

3.1. Tasks of the measurement

3.1.1. Measure the output voltage $U_2$ of the resistive divider consisting of ten resistors of the same value for all division ratios $d$ by means of:

   a) a DVM (digital voltmeter),
   b) a PMMC voltmeter (on 12 V measurement range).

Plot both the $U_2/U_1$ vs $d$ curves in a common graph and explain the differences. The input voltage of the divider ($U_1$) should be 10 V.

3.1.2. Calculate the divider output resistance $R_D$ from the measured values for the given division ratio $d$. Suppose the DVM input resistance be infinite.

3.1.3. Calculate the expanded uncertainty of type B (coverage factor $k = 2$) of the evaluated divider output resistance supposing that the input resistance tolerance of the PMMC voltmeter is less than 0.2 %.

3.2. Schematic diagram

![Schematic Diagram](image)

3.3. List of the equipment used

- **V** - PMMC voltmeter, accuracy class: ..., voltage range: ...
- **DV** - digital voltmeter, model: ..., accuracy ...
- **$U_1$** - DC power supply, model: ...

3.4. Theoretical background

If the output voltage of a resistive voltage divider is measured by a voltmeter the input resistance of which is comparable to the resistance of the divider, the output voltage $U_2$ is significantly lower than the corresponding output voltage of the unloaded divider $U_0$ (Fig. 3.2a).
The real divider powered from DC voltage source $U_1$ and loaded by voltmeter with input resistance $R_V$ according to Fig. 3.2b can be substituted (using Thevenin theorem) in series connection of DC voltage source $U_0$ and resistor $R_D$ according to the Fig. 3.2c.

Voltage $U_0$ is equal to the output voltage of unloaded divider and the resistance $R_D$ is given as parallel combination of divider resistors $R_1$ and $R_2$. Output divider voltage $U_2$ that is loaded by voltmeter input resistance $R_V$ is given as

$$U_2 = U_0 \frac{R_V}{R_V + R_D} \quad (3.1)$$

where $R_D$ is output divider resistance.

Voltmeter connection causes a methodical error of the following value

$$\Delta U_{\text{met}} = U_2 - U_0 = U_0 \frac{R_V}{R_V + R_D} - U_0 = U_0 \left( \frac{R_V}{R_V + R_D} - 1 \right) = U_0 \frac{-R_D}{R_V + R_D} \quad (3.2)$$

If the voltmeter input resistance is much higher than the output resistance of the measured voltage source (in our case output resistance $R_D$ of the voltage divider), the methodical error is insignificant. Therefore, using digital voltmeter (with input resistance $R_{DV} = 10^9 \, \Omega$) we measure voltage $U_{2DV}$ equal to the voltage value for unloaded divider, $U_0 \approx U_{2DV}$.

Different methods can be used to evaluate output resistance of the divider. One of them is based on schematics in Fig. 3.2c and relation (3.1), where $R_D$ can be evaluated. It gives

$$R_D = \frac{R_V(U_{2DV} - U_2)}{U_2} = R_V \left( \frac{U_{2DV}}{U_2} - 1 \right) \quad (3.3)$$

Another possibility is to start with loading characteristics of the divider (Fig. 3.3), where $I_2$ is the current passing through the load resistor $R_D$. Output characteristics is given as absolute value of slope of line:

$$R_D = \left| \frac{U_{2DV} - U_2}{0 - I_2} \right| = \frac{U_{2DV} - U_2}{U_2/R_V} = R_V \left( \frac{U_{2DV}}{U_2} - 1 \right) \quad (3.4)$$
**Output resistance measurement uncertainty evaluation**

If the fluctuations of measured values for each voltmeter are significantly lower than would correspond to its accuracy declared by its manufacturer (for PMMC voltmeter it is accuracy class, for digital multimeter it is the sum of errors in percent of range and of value), the A-type uncertainty is negligible and it is sufficient to measure voltages $U_{2DV}$ and $U_2$ (needed for resistance $R_D$ calculations) only once. In the opposite case, each measurement must be repeated several times, average values of $U_{2DV}$ and $U_2$ must be used instead and corresponding A-type uncertainties must be calculated (see [2]).

In our case where the A-type uncertainties of $U_{2DV}$ and $U_2$ measurements are negligible and the only sources of B-type uncertainty are inaccuracies of the used voltmeters and tolerance of the value of the input resistance of PMMC voltmeter.

The value of resistance $R_D$ can be calculated using (3.3). Since there is $R_D = f(U_0, U_2, R_V)$, uncertainty of the value of $R_D$ is

$$u_{R_D} = \sqrt{\left(\frac{\partial R_D}{\partial U_{2DV}} u_{U_{2CV}}\right)^2 + \left(\frac{\partial R_D}{\partial U_2} u_{U_2}\right)^2 + \left(\frac{\partial R_D}{\partial R_V} u_{R_V}\right)^2} \quad (\Omega) \quad (3.5)$$

where
- $u_{R_D}$ is resulting standard uncertainty of the $R_D$ ($\Omega$),
- $u_{U_{2DV}}$ standard uncertainty of measurement of $U_{2CV}$ (V),
- $u_{U_2}$ standard uncertainty of measurement of $U_2$ (V),
- $u_{R_V}$ standard uncertainty of value of $R_V$ ($\Omega$).

After evaluating the partial derivations

$$\frac{\partial R_D}{\partial U_{2DV}} = \frac{R_V}{U_2} \quad ; \quad \frac{\partial R_D}{\partial U_2} = -\frac{R_V}{U_2^2} \quad ; \quad \frac{\partial R_D}{\partial R_V} = \frac{U_{2DV} - U_2}{U_2} \quad (3.6)$$

we obtain the resulting standard uncertainty of the value of $R_D$

$$u_{R_D} = \sqrt{\left(\frac{R_V}{U_2} u_{U_{2DV}}\right)^2 + \left(\frac{R_V}{U_2^2} u_{U_2}\right)^2 + \left(\frac{U_{2DV} - U_2}{U_2} u_{R_V}\right)^2} \quad (\Omega) \quad (3.7)$$

Partial standard uncertainties are evaluated by the following way:

$$u_{U_{2DV}} = \frac{\delta_1 \times U_{2DV} + \delta_2 \times M_{DV}}{100 \sqrt{3}} \quad (V) \quad (3.8)$$

$$u_{U_2} = \frac{AC \times M}{100 \sqrt{3}} \quad (V) \quad (3.9)$$
\[ u_{RV} = \frac{\delta_{RV} \times R_V}{100 \sqrt{3}} \quad (\Omega) \quad (3.10) \]

where \( \delta_1 \) is error in percent of digital voltmeter reading,

\( \delta_2 \) error in percent of digital voltmeter full-scale range,

\( M_{CV} \) selected measurement range of the digital voltmeter (V),

\( AC \) accuracy class of the PMMC voltmeter (%),

\( M \) measuring range of the PMMC voltmeter (V),

\( \delta_{RV} \) resistor \( R_V \) tolerance (%).

Extended uncertainty \( U_{RD} \) of resistor \( R_D \) evaluation is obtained by multiplication of standard uncertainty by the coverage factor \( k \) (usually \( k = 2 \)), it means \( U_{RD} = k \, u_{RD} \).